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DIMENSIONAL CROSSOVER IN SUPERCONDUCTING SUPERLATTICES

Ivan K. Schuller*, Masashi Tachiki, and Earl Callen

With modern thin film vapor deposition techniques it is possible to construct superlattices of high compositional integrity, with little diffusion between layers. Layer thickness can be exactly controlled. Superlattices have been made with superconductors intersheaved with insulators, with semiconductors, with normal metals, with ferromagnets, and with other superconductors. At Yamada Conference XVIII on Superconductivity in Highly Correlated Fermion Systems, Sendai, Japan, 1987, one of us reviewed the properties of such superlattices. This article treats dimensional crossover in superconducting superlattices. Dimensional crossover describes a transition of the upper critical field parallel to the superlattice from the linear temperature dependence characteristic of three-dimensional bulk material to the square root dependence of thin two-dimensional films as the temperature is reduced. The cause of dimensional crossover is the contraction of the perpendicular coherence length to a size less than the nonsuperconducting layer thickness and a consequent uncoupling from each other of the superconducting layers in the superlattice.

INTRODUCTION

At Yamada Conference XVIII, Sendai, one of us discussed the superconducting properties of superlattices (Ref 1). For review articles see References 2 and 3. Thin films; sandwiches of superconductors with other superconductors, with normal metals, and with insulators; and multilayered and superlattice structures all have properties different from bulk, single-component materials.

The upper critical field H_{c2} of a bulk, isotropic superconductor is inversely proportional to the square of the coherence length. This produces a characteristic linear temperature dependence. In thin films there are two upper critical fields, H_{c2perp} and H_{c2par}.** When the field is perpendicular to the surface, persistent currents circulate in the film plane much as in the bulk, and H_{c2perp} is not greatly affected. But when the field is parallel to the surface, and when the coherence length ξ is greater than the film thickness d, H_{c2par} depends inversely as d^{2} rather than ξ^2. In superconducting superlattices, likewise, H_{c2perp} is as in bulk, single-component materials, but H_{c2par} has

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** Because of software limitations on typography in subscripts and superscripts we must use unconventional notation:

$$H_{c2perp} = H_{c2\perp} \text{ and } H_{c2par} = H_{c2\parallel}$$

$$\xi_{perp} = \xi_{\perp} \text{ and } \xi_{par} = \xi_{\parallel}$$

new features. In superlattices fabricated with thin insulating layers or with semiconductors, superconducting electron pairs, Cooper pairs, can tunnel through the intermediate layers. Josephson-coupled superlattices behave like bulk, single-component materials when the coherence length is large but like stacks of thin films when the coherence length is small compared to the layer thickness. Since coherence length changes with temperature, so does the temperature dependence of the upper critical field. Superlattices in which the alternate layers are normal metals are coupled not by Josephson tunneling but by the proximity effect; individual electrons travel between the superconducting layers. The result is once again that there are two temperature regimes of H_{c2per}. The theory of proximity-coupled superlattices has only recently been worked out.

BACKGROUND

Research on the properties of films was pioneered by Meissner (the Younger) (Ref 4), soon followed by Smith et al. (Ref 5), Rose-Innes and Serin (Ref 6), Simmons and Douglass (Ref 7), and by Hilsch (Ref 8). The early workers had to cope with interlayer diffusion, contamination, uneven film thickness, and voids. With the advent of sophisticated thin film vapor deposition techniques it became possible to create layered structures of a very high degree of geometric regularity. Consequently the theory now has firm data to explain.

Not that the theoreticians waited; they did not. Very early, Parmenter (Ref 9), Cooper (Ref 10), Douglass (Ref 11), de Gennes and Guyon (Ref 12), and Werthamer (Ref 13) laid down the framework of the theory. By 1964 de Gennes (Ref 14) was able to write a review article explaining the delicate matter of the appropriate boundary conditions.

A fundamental physical quantity that asserts itself in distinguishing thin from thick materials, and single component from heterogeneous structures, is the coherence length. Superconductivity results from electron pair correlation within a certain (temperature and mean free path dependent) distance, the coherence length. The superconducting wavefunction can extend into and through a normal metal. Transition temperatures, critical fields, and critical currents of thin superconducting films are reduced by contiguous normal metals, while the normal metals partake slightly of the superconducting properties. It is to be expected that the superconducting properties of the compound system will depend strongly on the thickness of the normal metal when that thickness is less than the coherence length, but the properties should be independent of normal metal thickness for thicknesses greater than the coherence length. And since the coherence length depends upon temperature, there can be two temperature regimes. In superconducting superlattices we shall see dramatic evidence of the transition between these regimes.

To understand the complicated behavior of layered structures we begin with a review of the relevant properties of single-component, bulk material. We need to understand coherence length, the Ginzburg-Landau (GL) equations, the temperature dependence of the coefficients in those equations, and the significance of the upper critical field. Fortunately, much of our work has been done for us; a previous issue of this *Scientific Information Bulletin* contains a review article on superconductivity (Ref 15). The GL equations were derived by minimizing the Gibbs free energy with respect to the vector potential A(r) and the quantum mechanical wavefunction ψ(r). In 1950 when Ginzburg and Landau proposed the theory, flux quantization (1961) had not yet been discovered, the BCS theory (1957) did not exist, and Cooper (1956) had not yet demonstrated the instability of the electronic state against the formation of Cooper pairs of electrons with oppositely aligned spins. The Ginzburg-Landau theory was a tour de force of insight. It was not clear what the wavefunction represented--the authors described it as an averaged superconducting electron wavefunction. To patch up the treatment now we consider the charge to be q = -2e. Although it is of no consequence, it is appealing to then normalize the wavefunction on one-half the number density of superconducting electrons and consider our pseudoparticle, or pair, to have a mass of 2m, noting that we have been overdefinite since what appears in the treatment is only the ratio ψ/m. de Gennes (Ref 16) comments that "we could just as well have chosen the mass of the sun." In Box 1 we give the GL equations and outline some of their consequences.

CRITICAL FIELD

There is a critical magnetic field H_c. When the external field exceeds H_c superconductivity is destroyed in a type I superconductor, to be defined below. The critical field falls quadratically with temperature from its maximum value at 0 K to zero at T_c. This is illustrated in Reference 15. There is also a critical current. In type I superconductors the critical current is that current which creates the critical field.

The critical field in type I superconductors (Hg, Sn, Al, Zn) is the field at which, if there were no geometric distortions of the magnetic field by shape effects, the superconductor would be transformed into the normal phase. A long cylinder parallel to an external field is converted entirely to the normal state when the field reaches H_c. For other shapes there are demagnetization effects and a gradual conversion over a range of fields. Over this shape-dependent range of fields the superconductor is in the "intermediate state." In the intermediate state the material is permeated by a fine, small-scale network of coexistent normal and superconducting regions. For type I superconductors a typical critical field is 500 gauss. The magnetic energy density, the depression of the superconducting energy below that of the normal state, is then $H_c^2/8\pi = 4 \times 10^4$ ergs/cm^3. This is to be compared with the Fermi energy of about 10^{11} ergs/cm^3. It is astonishing that some of the remarkable properties of superconductors--the critical temperature, the gap width--are rooted in a one-part-in-10^7 effect, and it is assuredly a tribute to the BCS theory (and to Nature, that has been kind to us again and made things simple where they could have been complicated!) that we are able to calculate those phenomena. On the other hand, other phenomena--flux quantization, Meissner effect, infinite conductivity--are a consequence purely of the spontaneous breakdown of electromagnetic gauge invariance and can be derived by a symmetry argument alone once that breakdown is assumed.

Box 1. The Ginzburg-Landau Equations

The first GL equation is the time-independent Schroedinger equation of a pseudoparticle in a magnetic field:

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{4m}\left| -i\hbar\nabla - \frac{2e}{c}A\right|^2\psi = 0 \qquad \text{(B1-1)}$$

It contains an unusual cubic term but can be looked on as a self-consistent Schroedinger equation in which the energy is

$$E = -(\alpha + \beta|\psi|^2) \qquad \text{(B1-2)}$$

One can, of course, generalize Equation B1-1 to a time-dependent form, and this is often done.

The significance of the coefficients α and β in Equation B1-1 is found by considering a situation with no currents or magnetic fields and when the wavefunction is constant. The two solutions of Equation B1-1 are $\psi = 0$, the normal state, and $\psi = \psi_o$. With n_s the density of superconducting electrons,

$$|\psi_o|^2 = -\frac{\alpha}{\beta} = \frac{n_s}{2} > 0 \qquad \text{(B1-3)}$$

This is the superconducting solution. It will be the lower energy solution when $\alpha/\beta < 0$. We shall return to this when we consider temperature dependence. Equating the energy difference between the normal and superconducting phases to the magnetic energy required to destroy superconductivity one finds

$$\frac{\alpha^2}{2\beta} = \frac{H_c^2}{8\pi} \qquad \text{(B1-4)}$$

(H_c is the critical field. See Reference 14 and the CRITICAL FIELD section.) The dimensions of α are energy per particle and of β are energy times volume per particle squared:

$$\alpha = \frac{-H_c^2}{2\pi n_s} \qquad \text{(B1-5)}$$

and

$$\beta = \frac{H_c^2}{\pi n_s^2} \qquad \text{(B1-6)}$$

continued

Box 1. The Ginzburg-Landau Equations (continued)

The scale of Equation B1-1 is the coherence length ξ. Consider a situation with no currents or magnetic fields but in which the wavefunction need not be uniform, so there is a kinetic energy associated with its curvature. The scale of "stiffness" is set by

$$\xi^2(0) = \frac{-\hbar^2}{4m\alpha} = \frac{\pi\hbar^2 n_s}{2mH_c^2} \qquad \text{(B1-7)}$$

There is another length scale. The third term in Equation B1-1 reproduces the electromagnetic properties of the London equation and gives the London penetration depth λ, the scale of decay of a static magnetic field into a superconductor because of screening by induced persistent currents (Ref 15).

By means of Equation B1-1 the concept of interface energy can be developed (Ref 15). It turns out (see the following section on upper critical field H_{c2})that there are two regimes of behavior and classes of materials, depending upon whether the interface energy in a layer at the surface of a superconductor is positive or negative and distinguished by the important Ginzburg-Landau parameter

$$\kappa = \lambda/\xi \qquad \text{(B1-8)}$$

The second GL equation describes the current density:

$$j = \frac{-2e\hbar}{4im}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{4e^2}{2mc}|\psi|^2 A \qquad \text{(B1-9)}$$

If there is no external field and the wavefunction is $(n_s/2)^{1/2}$ exp (ikx), the current density is (-2e) (v) $(n_s/2)$.

UPPER CRITICAL FIELD H_{c2}

Imagine a metal to fill space, but in a uniform magnetic field so strong as to suppress superconductivity. The magnetic field is now reduced to that strength at which the metal just becomes superconducting. The wavefunction will then be weak and we can neglect the cubic term in Equation B1-1. Neglecting the field due to the supercurrent and identifying the vector potential as that of the external field H, Equation B1-1 is the Schroedinger equation of our pseudoparticle in a uniform external field. Classically the particle rotates around the field at the cyclotron frequency

$$\omega_c = \frac{(2e)H}{(2m)c} \qquad (1)$$

Quantum mechanically the allowed levels are the Landau levels, of energy

$$E_n = (n + 1/2)\hbar\omega_c$$

$$+ (1/2)(2m)v^2 \text{ longitudinal} \quad (2)$$

The lowest energy, the n=0 level (and with zero velocity parallel to the magnetic field), occurs at the upper critical field, the field strength at which flux begins to penetrate:

$$-\alpha = (1/2)\hbar\omega_c = \frac{\hbar e H_{c2}}{2mc} \quad (3)$$

Combining this result with the definition of κ (Equation B1-8) and using the relations in Box 1, one arrives at

$$H_{c2} = \sqrt{2}\,\kappa\, H_c \quad (4)$$

By this result we can understand the dichotomy into type I and type II materials alluded to previously. Suppose $\kappa > 1/\sqrt{2}$, so that $H_{c2} > H_c$. As the field is lowered below H_{c2}, field lines enter, and yet the material cannot be completely normal because $H > H_c$. The material is in a mixed state. This is type II behavior. On the other hand, if $\kappa < 1/\sqrt{2}$, $H_c > H_{c2}$. As one reduces the external field, at H_c the material becomes completely superconducting. Above H_c the material was normal; below H_c there is a complete Meissner effect. H_{c2} has no physical meaning. This is type I behavior. The results in Box 1 can also be combined in another form that we shall find useful:

$$H_{c2} = \frac{\Phi_o}{2\pi\xi^2} \quad (5)$$

(Φ_o is the flux quantum, the smallest allowed bundle of magnetic flux. Its value is ch/2e, about 2.1×10^{-7} gauss x cm^2. See Reference 15.) See Box 2 for a discussion of type I and type II superconductors.

Box 2. Type I and Type II Superconductors

Type I Superconductors. $\kappa < 1/\sqrt{2}$. Zn, Cd, Hg, Al, Ga, In, Tl, Sn, Pb.

In simple metals of broad bandwidth the effective mass is small and the Fermi velocity is large. These metals exhibit a Meissner effect, but the field does not drop off exponentially. There are nonlocal effects due to the "stiffness" of the condensed phase wavefunction over a coherence length.

Type II Superconductors. $\kappa > 1/\sqrt{2}$. Nb, V, Nb_3Sn, Nb_3Ge, V_3Ga, V_3Si, MoRe alloys, some Pb alloys, the new high T_c copper ceramics.

In transition metals and in intermetallic compounds and oxides of narrow bandwidth the effective mass is large, and hence λ is large (> 2×10^{-5} cm). At the same time the Fermi velocity is small ($\approx 10^6$ cm/s) and the energy gap and transition temperature are large (Nb_3Ge has a transition temperature of 22.3 K), so ξ is small (see Ref 15). In the ceramic copper oxides the coherence length is only about 1 nm. Also in disordered alloys, because the coherence length is reduced with the mean free path by the scattering, the London approach is applicable.

CRITICAL FIELDS IN TYPE II SUPERCONDUCTORS: H_{c1}, H_{c2}, H_{c3}

Recall that a long cylinder of type I material in a magnetic field is transformed all at once from superconducting to normal by the critical field H_c. This field strength is a measure of the difference of free energy densities of the normal and superconducting phases:

$$g_n - g_s = H_c^2/8\pi \qquad (6)$$

In type II materials one can consider Equation 6 to be a definition of the critical field strength. The free energy difference can be measured by other means, such as specific heat, and H_c determined. One finds the following phenomena when a type II rod is placed in a longitudinal field:

1. There is a "lower critical field," H_{c1}, less than H_c, below which the Meissner effect is complete--there is no flux penetration into the sample.

2. As H exceeds H_{c1} magnetic flux begins to penetrate the sample. The field lines induce persistent currents. The material is superconducting but with an incomplete Meissner effect. With increasing external field more field lines enter the sample until an "upper critical field," H_{c2}, is reached. H_{c2} exceeds the theoretical H_c, sometimes by a very large factor. It is this that makes type II materials useful. At H_{c2} the flux penetration is complete and the bulk material is normal. Figure 1 illustrates the induction B as a function of applied longitudinal field strength H in a long, thin wire of type I and type II superconductors. Figure 2 shows $-4\pi M$ [$= -(B-H)$] versus H. The integral of M dH is the magnetic energy density stored in the superconductor. It can be shown by a thermodynamic argument that if type I and type II superconductors have the same H_c--the same superconducting energy--the areas under their magnetization curves are the same.

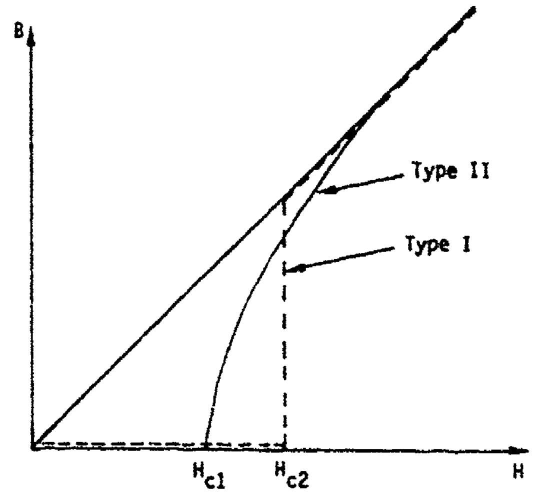


Figure 1. Induction B versus external field H curves for ideal type I and type II superconducting rods in a longitudinal field. The dashed line represents type I behavior and the solid line type II.

3. Above H_{c2}, though bulk superconductivity is gone, if the field is parallel to the surface there remains a surface superconducting layer of thickness $\xi(T)$ up to field strength H_{c3} (Ref 17). H_{c3} = 1.69; $H_{c2} = 2.4\ \kappa\ H_c$. Critical fields are at their maximum at T = 0 K, fall monotonically with increasing temperature, and vanish at T_c. Figure 3 is an (H,T) phase diagram of a long rod of type II superconductor in a longitudinal field.

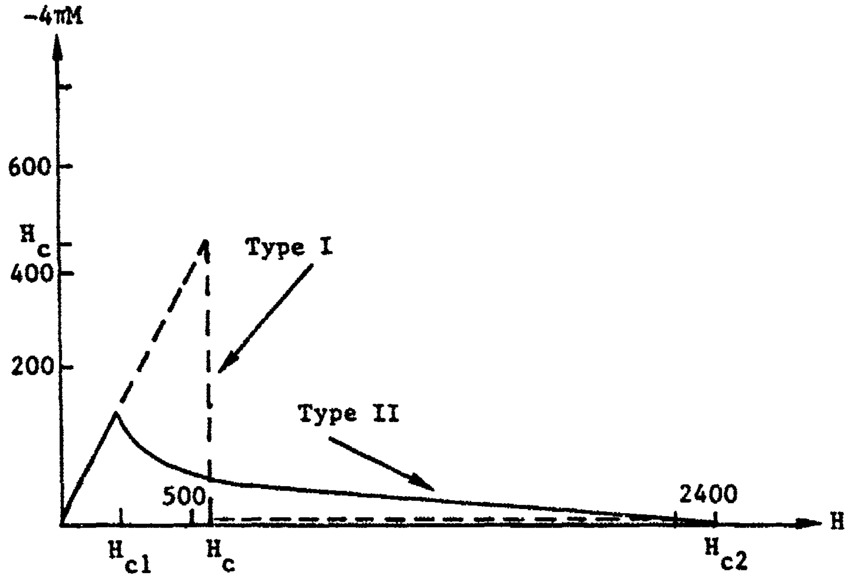


Figure 2. Magnetization versus field curves for type I (dashed) and type II (solid line) superconductors. It can be shown that if the two materials have the same H_c, the areas under the two curves are equal.

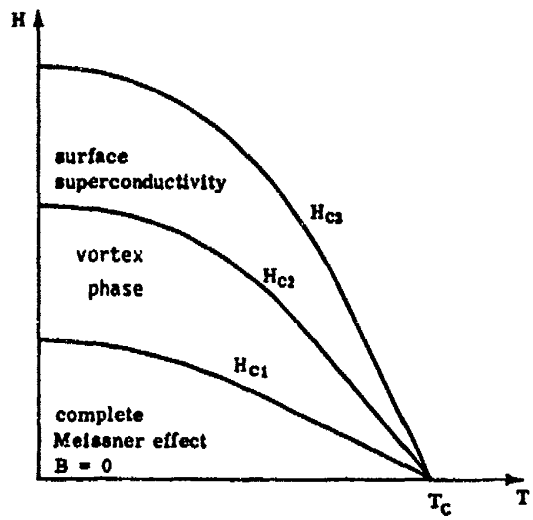
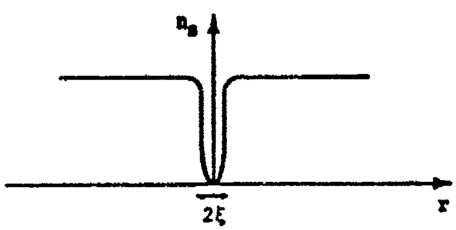


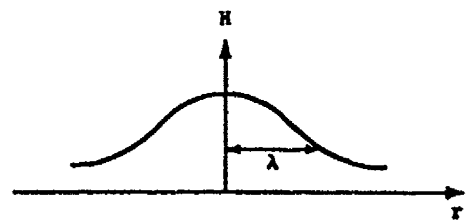
Figure 3. (H,T) phase diagram for a type II superconductor. All three critical fields fall with temperature and vanish at T_c. In the phase below H_{c1} the field is excluded from the sample (on the macroscopic scale; there is a thin penetration layer at the surface in which it falls to zero). Between H_{c1} and H_{c2} field lines increasingly penetrate the material. This is the vortex phase. At H_{c2} there is complete penetration of the field into the bulk. Bulk superconductivity is suppressed by the field, but a superconducting surface layer remains up to field strength H_{c3}.

VORTICES

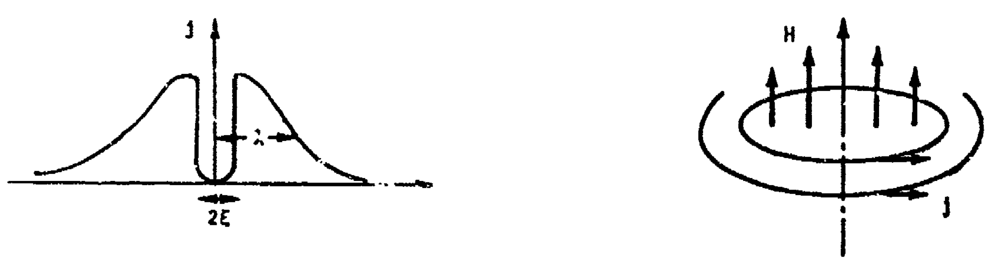
Between H_{c1} and H_{c2} individual filaments of normal material surrounded by magnetic field lines and current vortices enter the superconductor. The structure of these vortices is understood, particularly in the large λ limit; it is illustrated in Figure 4. Each filament has a narrow (radius ξ) core of normal material. The magnetic field is confined to a range λ around the core, and its strength falls off exponentially from the axis of the core. The field decays with distance because it is screened by a circulating persistent current rotating cylindrically around the core. The filaments are repulsive; they minimize their interaction energy by arranging themselves in a triangular array. (Recall the pores in an anodic oxidized aluminum plate; Ref 18.)



(a) At the axis of the filament is a core of normal material of radius ξ.



(b) The magnetic field is confined to a cylindrical region of radius λ; its strength decreases exponentially with distance from the axis.



(c) and (d) The field drops off because of screening by a vortex of persistent current. Outside the normal core this current also falls off exponentially with distance from the axis.

Figure 4. A magnetic field filament in a type II superconductor in the vortex state between H_{c1} and H_{c2}. (Adapted from Y.B. Kim, Physics Today, September 1964, pp. 21-30).

TEMPERATURE DEPENDENCE

GL theory is the application to superconductivity of the Landau theory of second-order phase transitions. It is particularly well suited to type II superconductors. The Landau theory (Ref 19) assumes analyticity of the free energy through the second-order phase transition. The theory emphasizes that there is an order parameter that goes to zero continuously at the transition and assumes that the free energy can be expanded near the transition in powers of this order parameter.* The realm of validity of the theory should then be in the neighborhood of the phase transition, where the order parameter is small. And so we must introduce temperature. Return to Equation B1-1 and the discussion following it. When $T < T_c$ one wants the superconducting solution to lie lower in energy; that is, one wants $\alpha/\beta < 0$. When $T > T_c$ one wants the normal $\psi = 0$ solution to lie lower; $\alpha/\beta > 0$. In order to keep the solution bounded one wants β positive. The simplest way to effect this is the assumption introduced by Landau: one sets α proportional to $(T-T_c)$. The coefficient β is presumed to have a negligibly weak temperature dependence compared with α and is simply taken as constant. The previously introduced $\lambda(0)$ and $\xi(0)$ are the $T = 0$ values of these quantities. This forces the temperature dependence of the other quantities. The program is carried out in Box 3.

ANISOTROPIC EFFECTS, FILMS

One of the earliest suggestions for dealing with bimetal composite films was that of Cooper (Ref 10). Cooper argued that because correlation introduces a nonlocality into the electron pair wavefunction the effective interaction should be a spatial average of the attractive potentials on the two sides of the interface. de Gennes (Ref 14) showed that averaging is appropriate only for d_S and d_N both much less than the coherence length, when spatial variation of the order parameter can be neglected. In that case the "effective NV," the interaction energy to be used in the BCS formula for the transition temperature of a bimetal film, is not exactly the simple spatial average Cooper suggested, but is similar to it.

*The Landau theory of second-order phase transitions also does much more. It is a symmetry theory. Landau recognized that the symmetry group of the high-temperature disordered phase is of larger order than that of the low-temperature ordered phase; the symmetry group in the ordered phase is a subgroup of that above the transition. The Landau theory rests on the assumption that the free energy is analytic at the phase transition; in point of fact it is not. Relatively large fluctuations in macroscopic parameters alter the behavior from that of classical mean field theory to the "critical exponents" of renormalization group. In magnetism, particularly in systems of low spin quantum number and short range exchange interaction, this leads to short range order persisting far into the paramagnetic regime. But in superconductivity the interaction has long-range components, and deviations from mean field theory behavior are not so large as to invalidate it, except in a narrow temperature range at the transition, and only for those quantities sensitive to short range order, such as the specific heat.

Box 3. Temperature Dependence

We assume that β is a positive constant, to assure boundedness of the solution, and that α is linear in the deviation of T from T_c:

$$\alpha = \alpha' \left(\frac{T - T_c}{T_c} \right) \tag{B3-1}$$

Then from Equation B1-7,

$$\alpha' = \frac{\hbar^2}{4m\xi^2(0)} \tag{B3-2}$$

$$\begin{aligned} n_s(T) &= n_s(0) \left(\frac{T_c - T}{T_c} \right); \quad T < T_c \\ &= 0 \qquad\qquad\qquad\quad T > T_c \end{aligned} \tag{B3-3}$$

From Equation B1-5,

$$H_c(T) = H_c(0) \left(\frac{T_c - T}{T_c} \right) \tag{B3-4}$$

Only near T_c does Equation B3-4 conform to observation, which is better fitted (Ref 20) by

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c} \right)^2\right] \tag{B3-5}$$

These are not so different; for small $[(T_c - T)/T_c]$ the empirical relation is approximately

$$H_c(T) \simeq 2H_c(0) \left(\frac{T_c - T}{T_c} \right)$$

The temperature dependent penetration depth becomes

$$\lambda(T) = \lambda(0) \left(\frac{T_c}{T_c - T} \right)^{1/2} \tag{B3-6}$$

continued

Box 3. Temperature Dependence (continued)

The temperature dependent correlation length becomes

$$\xi(T) = \xi(0)\left(\frac{T_c}{T_c - T}\right)^{1/2} \qquad (B3\text{-}7)$$

Note that κ is independent of temperature as it should be; materials do not switch from type I to type II with changing temperature.

Lastly, we have the important relations

$$H_{c2}(T) = \sqrt{2}\kappa H_c(0)\left(\frac{T_c - T}{T_c}\right) \qquad (B3\text{-}8)$$

$$H_{c2}(T) = \frac{\Phi_o}{2\pi\xi^2(T)} = \frac{\Phi_o(T_c - T)}{2\pi\xi^2(0)T_c} \qquad (B3\text{-}9)$$

Saint-James and de Gennes (Ref 17) described surface superconductivity. When a magnetic field perpendicular to the surface is decreased from a high value in a potentially superconducting material, the material becomes superconducting in the vortex state at the same field (Equation B3-9) as in bulk material. But when the field is parallel to the surface, the nucleation field H_{c2par} is increased because of the suppression of circulating currents (see the section on critical fields in type II superconductors discussed previously). Following these ideas, Werthamer, Helfand, and Hohenberg (Ref 21) calculated the upper critical field in the dirty limit (correlation length greater than mean free path).

In thin (quasi-two-dimensional) superconducting films whose thickness d is smaller than the correlation length, the upper critical fields are given, according to Tinkham (Ref 22), by

$$H_{c2perp} = \frac{\Phi_o}{2\pi\xi^2(T)} = \frac{\Phi_o}{2\pi\xi^2(0)}\left(\frac{T_c - T}{T_c}\right) \qquad (7)$$

$$H_{c2par} = \frac{\Phi_o}{2\pi\xi(T)d/\sqrt{12}} = \frac{\Phi_o}{2\pi\xi(0)d/\sqrt{12}}\left(\frac{T_c - T}{T_c}\right)^{1/2} \qquad (8)$$

SUPERLATTICES

Dimensional Crossover

The first calculations of the upper critical fields of superconductor superlattices (Ref 3) were by Kats (Ref 23); Lawrence and Doniach (Ref 24); and later by Klemm, Beasley, and Luther (Ref 25) and Deutscher and Entin-Wohlman (Ref 26). H_{c2} was found through application of the anisotropic Ginzburg-Landau equations. Multilayered compounds were modelled as a stack of two-dimensional superconductors, with no variation in the order parameter across each layer, and the layers coupled via Josephson tunneling. The result can be expressed in terms of two coherence lengths, $\xi_{par}(T)$ and $\xi_{perp}(T)$.

The less interesting geometry is when the external field is perpendicular to the layers, for then the orbital currents that circulate in the vortices within each layer do not sense layer thickness. The situation is much like that of a magnetic field perpendicular to the surface of a homogeneous bulk material: the upper critical field is linear in the temperature (Equation B3-9).

When the field is parallel to the surface new things can happen, depending upon the thicknesses of the superconducting and the normal (insulating, semiconducting, or metal) layers and the temperature. Thin ($dN << \xi_{perp}$) layers allow the superconducting layers to couple by Josephson tunneling (insulating or semiconducting layers) and by the proximity effect (metal layers) (Ref 27), and three-dimensional behavior is observed. The Lawrence and Doniach theory (Ref 24) applies:

$$H_{c2perp} = \frac{\Phi_o}{2\pi\xi^2(T)} = \frac{\Phi_o}{2\pi\xi^2(0)}\left(\frac{T_c-T}{T_c}\right) \qquad (9)$$

$$H_{c2par} = \frac{\Phi_o}{2\pi\xi_{perp}(T)\xi_{par}(T)} = \frac{\Phi_o}{2\pi\xi_{perp}(0)\xi_{par}(0)}\left(\frac{T_c-T}{T_c}\right) \qquad (10)$$

But suppose the thickness of the normal layers significantly exceeds the T = 0 perpendicular correlation length of the superconductor [$d_N > \xi_{perp}(0)$] and the thickness of the superconducting layers is less than the correlation length [$d_s < \xi_{perp}(0)$]. A phenomenon known as "dimensional crossover" then occurs. At low temperatures the superconducting layers are uncoupled; two-dimensional square root dependence (Equation 8) is observed. As the temperature is increased, $\xi_{perp}(T)$ grows larger than d_N; many superconducting layers are coupled and three-dimensional linear temperature dependence (Equation 10) results.

Dimensional crossover has been observed in naturally occurring intercalated transition metal dichalcogenides (Ref 28) and in artificially grown superlattices--superconductor/insulator Nb/Al_2O_3 (Ref 29); superconductor/semiconductor Nb/Ge (Ref 30), Mo/Si

(Ref 31), Pb/Ge, Pb/C (Ref 32); superconductor/metal Nb/Cu (Ref 27), V/Ag (Ref 33), Nb/Ti (Ref 34), Nb/Ag (Ref 35), Nb/Ta (Ref 35), NbTi/Ti (Ref 36); superconductor/ferromagnet V/Ni (Ref 37); and superconductor/superconductor Nb-Ti/Nb (Ref 36, 38).

In earlier work surface superconductivity obscured the results. Surface conductivity can be suppressed by depositing sufficiently thick (≈300 nm) coatings of a good normal conductor such as copper on the outermost surfaces. Figure 5 shows standard three-dimensional behavior in a thick (850 nm) Nb film. Perpendicular and parallel upper critical fields are the same; they decrease linearly with increasing temperature up to the critical temperature. In Figure 6 we show (Ref 1) the two critical fields in a Nb layer whose thickness (19 nm) is less than the bulk correlation length ($\xi_{Nb} \approx 40$ nm). While the perpendicular critical field shows no indication of reduced dimensionality, the parallel critical field displays typical two-dimensional character. Figure 7 shows dimensional crossover. Within the thick outer copper layers there is a superlattice of alternately Nb (17.1 nm) and Cu (37.6 nm). At all temperatures H_{c2perp} varies as $(T_c\text{-}T)$, since $\xi_{par}(T)$ is not affected by dimensional change (Ref 32). On the other hand, at lower temperatures, when ξ_{perp} is smaller than d_{Cu} but exceeds d_{Nb}, H_{c2par} varies as in a two-dimensional film. But as the temperature increases ξ_{perp} exceeds d_{Cu}, the superconducting layers are coupled together, and the temperature dependence is linear (Ref 1).

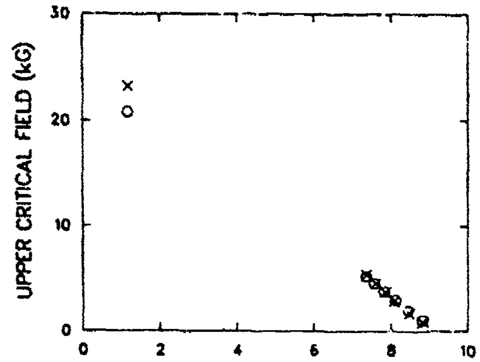


Figure 5. Upper critical fields versus temperature of a thick niobium film. The thickness of the Nb layer (850 nm) greatly exceeds the Nb correlation length (≈ 40 nm). The single layer of Nb is covered with thick (300 nm) layers of Cu to suppress surface superconductivity. The upper critical fields show typical three-dimensional behavior; H_{c2perp} and H_{c2par} are equal and fall linearly with increasing T up to T_c.

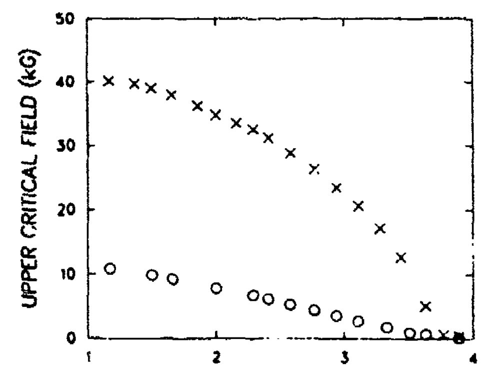


Figure 6. Upper critical fields versus temperature of a thin niobium film. Within the outer thick Cu layers is a single sheet of Nb whose thickness (19.1 nm) is less than the Nb correlation length (≈ 40 nm). This is a typical two-dimensional situation; H_{c2par} exceeds H_{c2perp}. H_{c2perp} varies linearly with $(T_c\text{-}T)$, but H_{c2par} falls with increasing temperature as $(T_c\text{-}T)^{1/2}$.

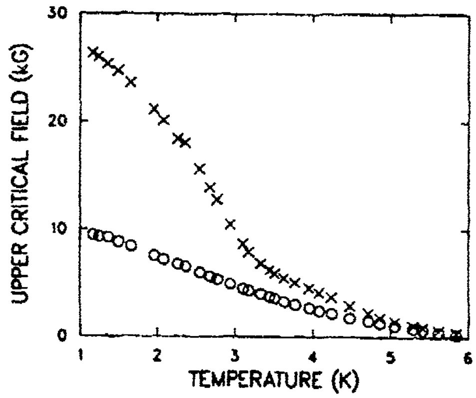


Figure 7. Dimensional crossover in a niobium/copper superlattice. The thickness of the Nb layers (17.1 nm) is less than the T=0 Nb correlation length (≃ 40 nm). The Cu layer thickness (37.6 nm) in the superlattice is about the same as the Nb correlation length. H_{c2perp} varies as (T_c-T) at all temperatures. At low temperatures the Nb layers are uncoupled, and H_{c2par} depends on temperature as the square root. But at higher temperatures ξ_{Nb} greatly exceeds d_{Cu}, the layers couple together, and linear temperature dependence results.

Proximity Effects, Ferromagnetism, Complications

To this point we have been able to employ only an appealingly simple theory, an effective mass theory, in which the order parameter is treated as uniform in the superconducting layers, and with the layers coupled only by Josephson tunneling (Ref 24). But when layer thickness exceeds the coherence length the order parameter is nonuniform. Proximity effects must be taken into account. Density of states at the Fermi level, electron diffusivities, mean free paths, Debye temperatures, all can be expected to differ in the materials constituting the superlattice. As is to be expected, as the theory has matured complications have been progressively attacked and included and new experimental features have been discovered and analyzed.

The most complete and detailed treatments are those of Tachiki and Takahashi (Ref 39) and of Biagi et al. (Ref 40) on the perpendicular upper critical field. Tachiki and Takahashi extend the treatment of de Gennes and Guyon (Ref 12) and de Gennes (Ref 14) to include proximity effects through normal metal layers. They allow the densities of states to differ in the two components, as well as the conduction electron diffusion constants. To treat superconductor/magnetic superlattices Tachiki and Takahashi include the effect of spin polarization of conduction electrons in the ferromagnetic layers.

There are four kinds of superconductor superlattices to be considered: Josephson coupled, in which the normal material is an insulator (Ref 29) or a semiconductor (Ref 30-32); proximity coupled, in which the normal layers are metallic (Ref 27, 33-36); magnetic (Ref 37); and superconductor/superconductor (Ref 34, 36, 38).

Dimensional crossover was understood for Josephson-coupled superlattices, and Schuller had proposed that the same phenomenon would occur in proximity-coupled superlattices. Tachiki and Takahashi (Ref 39) have calculated this. They find the same qualitative behavior as in Josephson-coupled superlattices. Their theory is now confirmed in quantitative detail in Nb/Cu (Ref 1, 27) and by the measurements of Kanoda et al. (Ref 33) on V/Ag, Nakajima et al. (Ref 34) on Nb/Ti, and Ikebe et al. (Ref 35) on Nb/Ag and Nb/Ta. Dimensional crossover progresses systematically with the ratio of the density of states of the superconductor and the separator.

In all experiments on superlattices up to this point the parallel upper critical field was larger than the perpendicular upper critical field. In superconductor/magnetic superlattices in a temperature range just below T_c, H_{c2par} is less than H_{c2perp}. This has been observed by Homma et al. (Ref 37) in V/Ni superlattices and explained, at least qualitatively, by Takahashi and Tachiki (Ref 39). Magnetization measurements show that the conduction electron spin polarization in the Ni layers is highly anisotropic; the spin polarization induced by a magnetic field parallel to the layer is far larger than that induced by a perpendicular field. Thus, the pair breaking field due to the spin polarization is greater when the external field is parallel to the layers than when it is perpendicular. At high temperatures near T_c the coherence length is large, the vortices extend over many layers, and H_{c2par} is more greatly reduced by the pair breaking field in the nickel than is H_{c2perp}. But as the temperature is reduced the coherence length shrinks to less than the layer thickness and the vortices lie within the nickel layers in the parallel field configuration. The situation is then not very different from that in a nonmagnetic superlattice: H_{c2par} again exceeds H_{c2perp}.

Superconductor/superconductor superlattices merit particular attention. Takahashi and Tachiki pointed out that another source of dimensional crossover is possible when the two components have different diffusion constants D. Just below T_c the coherence length is large and many layers are coupled. Other things being equal, the order parameter nucleates preferentially in the set of layers of large D. The low diffusion constant of the dirty layers tends to confine the pair potential to the high D layers. At lower temperatures the order parameter shifts to the dirty layers and is confined there by the short coherence length. The layers uncouple. Obi et al. (Ref 36) observed a break in slope of H_{c2par} in NbTi/Nb.

The measurements of Karkut et al. (Ref 38) on NbTi/Nb superconducting superlattices also conform to the Takahashi and Tachiki theory and to plausible expectation. The thicker the layers the higher the temperature at which layer thickness exceeds coherence length, and the higher the temperature at which the break in slope should occur. This is shown in Figure 8, from Karkut et al. (Ref 38).

Dimensional crossover can manifest itself in another way. While H_{c2perp} is insensitive to layer thickness, H_{c2par} is enhanced when ξ_{perp} is less than the layer thickness. The T = 0 K anisotropy ratio, $H_{c2par}(0)/H_{c2perp}(0)$, should then peak at a thickness something like the coherence length. For simplicity consider $d_s = d_N = d$. Figure 9 is from Takahashi and Tachiki (Ref 39). Layer thickness d is plotted in units of coherence length. Height and position of the maximum shifts to lower d/ξ for lower ratios of normal to superconducting density of states. Banerjee et al. (Ref 27) observed a sharp peak in H_{c2par}/H_{c2perp} in Nb/Cu superlattices. Figure 10 is from Obi et al. (Ref 36). It shows the low temperature (1.5 K) ratio of the two upper critical fields measured on two NbTi/Ti multilayers. One of these, designated NTT1, is $Nb_{65}Ti_{35}/Ti$; the other, NTT2, is $Nb_{28}Ti_{72}/Ti$. The abscissa is the modulation wavelength of the superlattice ($d_s = d_N = \Lambda/2$). The authors interpret the shift to a higher peak at lower layer thickness as evidence for a higher density of states in the superconducting layers of the more Nb-rich material (a lower N_N/N_S), in support of the Takahashi and Tachiki theory. But at the same time, measurements (Ref 36) on NbTi/Nb fail to conform to the theory.

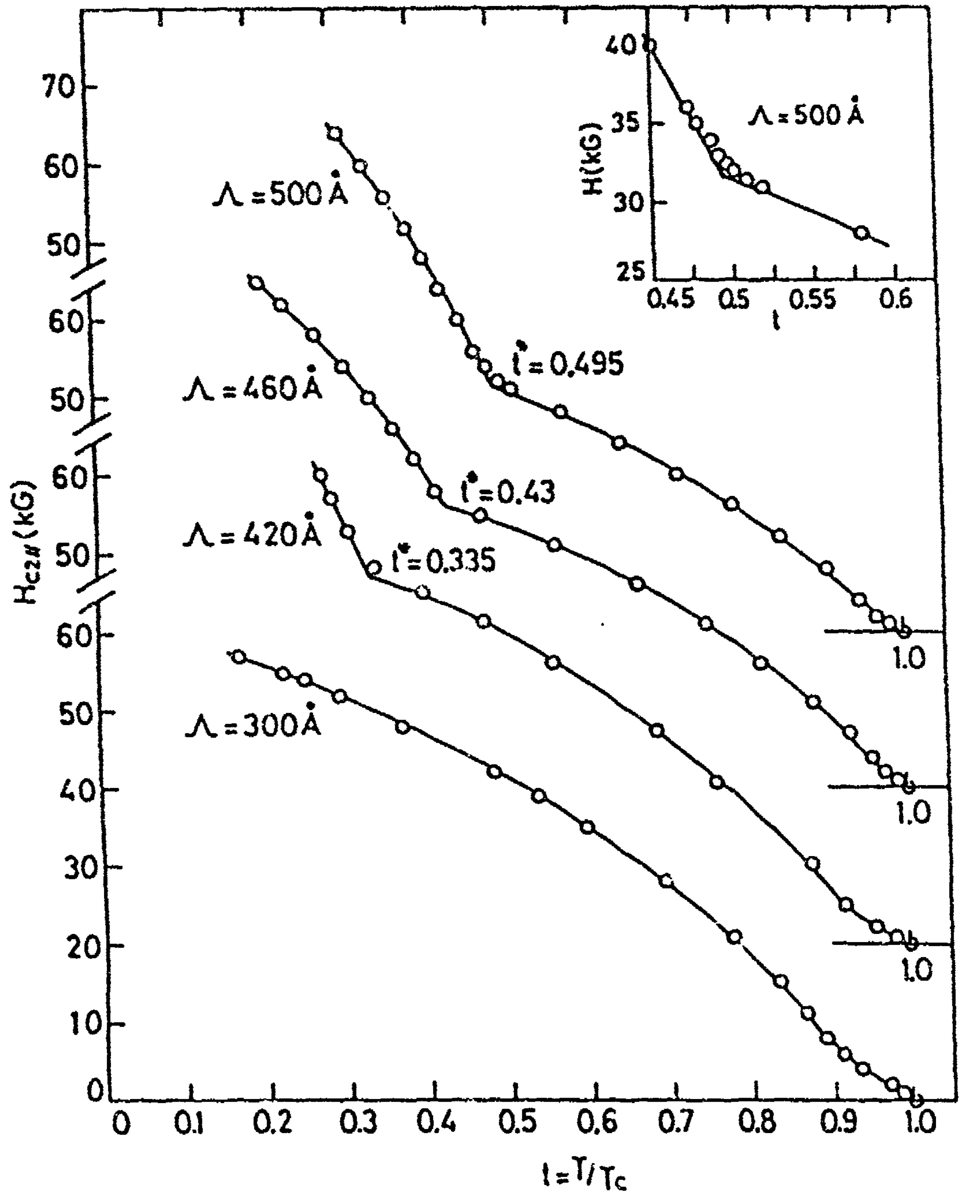


Figure 8. H_{c2par} versus t (= T/T_c). Dimensional crossover in a superconductor/superconductor superlattice. The two components, $Nb_{65}Ti_{35}$ and Nb, have approximately the same T_c; they differ principally in electron diffusion constants. D of the disordered alloy is expected to be much less than that of the pure metal ($D_{Nb}Ti/D_{Nb}$ = 0.048). $\Lambda = d_{NbTi} + d_{Nb}$. t* (= T^*/T_c) is the reduced temperature at which the break in slope occurs. The y axes are displaced for clarity and the solid lines are guides to the eye. Blowup for the region around t* for the Λ = 500 Å sample (from Ref 38).

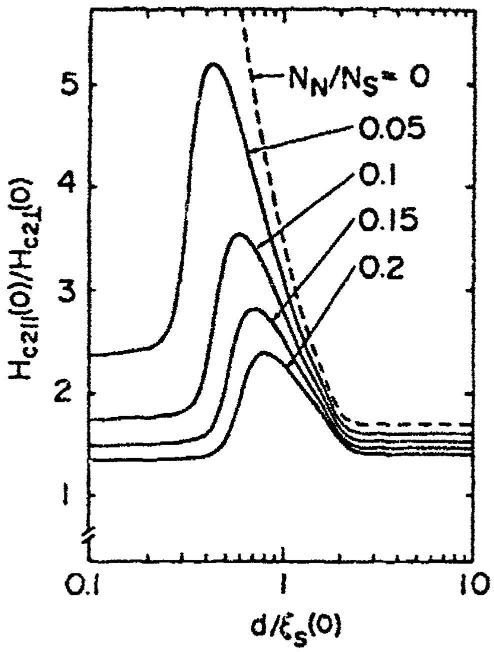


Figure 9. Ratio of H_{c2par} to H_{c2perp} at $T = 0$ K as a function of layer thickness $d = d_N = d_S$. Thickness is normalized on the coherence length $\xi_S(0)$. Curves show several ratios of densities of states at the Fermi level of the normal and superconducting materials. The figure is from Reference 39 and the dashed curve conforms to Reference 17.

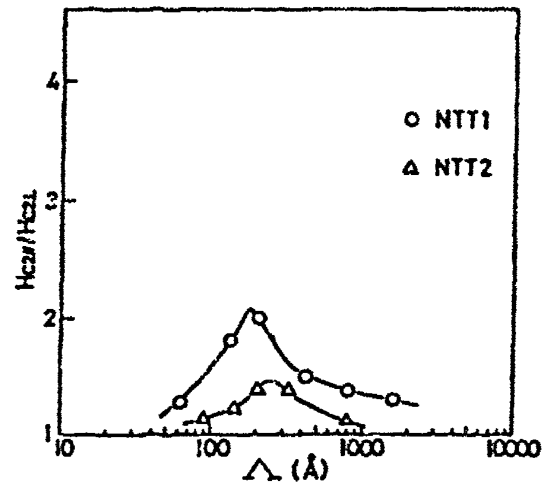


Figure 10. Anisotropy ratio of upper critical fields, H_{c2par}/H_{c2perp}, as a function of superlattice repeat distance ($\Lambda = 2d$; $d = d_S = d_N$) at 1.5 K. NTT1 = $Nb_{65}Ti_{35}/Ti$; NTT2 = $Nb_{28}Ti_{72}/Ti$. NTT1 presumably has a higher density of states at the Fermi level, N_S (and therefore a lower N_N/N_S), than NTT2 (from Ref 36).

CERAMIC SUPERLATTICES: THE FUTURE

How about high T_c ceramic superconductor superlattices--with each other, with conventional superconductors, with normal metals and semiconductors? About the only thing we know about high T_c superlattices is that they will be made. Tonouchi et al. (Ref 41) of Osaka University have fabricated up to five alternate layers of yttrium-barium-copper oxide, erbium-barium-copper oxide, and others. Multilayers with 1.5 nm thick layers and with 60 nm layers have been produced. Depending upon the oxygen defect concentration, the ceramic oxides have the electronic transport properties of insulators, semiconductors, metals, or superconductors. One of their many mysteries is that though they do not look metallic--they are not shiny--they have metallic, linear temperature dependence of resistivity over a very wide temperature range above T_c. Who knows what electronic devices it will be possible to make from ceramic superlattices? The coherence length in the oxides is extremely short--about 1 nm, a lattice constant. This must change the physics. What happens in contiguous superconductors with different mechanisms for Cooper pair formation? The Josephson effect has been observed. How about proximity effects and dimensional crossover--are they different? The future is surely full of surprises.

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Ivan K. Schuller is Professor of Physics at the University of California at San Diego and a Special Term Appointee at Argonne National Laboratories. He received his Licenciado of Ciencias (1970) from the University of Chile and his M.S. (1972) and Ph.D. (1976) degrees from Northwestern University. Professor Schuller is coauthor of more than 160 publications and patents and has given numerous invited talks at national and international meetings. Professor Schuller is a Fellow of the American Physical Society, recipient of the 1981 University of Chicago "Medal for Distinguished Performance at Argonne National Laboratory," and was awarded the 1981 "Technology 100" and the 1987 Department of Energy "Outstanding Scientific Accomplishment in Solid State Physics" awards. Dr. Schuller has been active in popularizing condensed matter physics among nonspecialists. He is especially interested in promoting international scientific relations, as Secretary/Treasurer of the International Physics Group of the American Physical Society.

Masashi Tachiki is Professor of Physics at Tohoku University and a member of their Institute for Materials Research. He received his education at Okayama and Osaka Universities, and was an Associate Professor at Osaka University before moving to Sendai. He has published 138 research articles and contributed to or edited 4 books on the superconducting and magnetic properties of solids. Professor Tachiki has held many visiting appointments at American universities. He has been a member of the Organizing Committee or the Advisory Committee of a large number of national and international conferences.

Earl Callen is a member of the staff of the Office of Naval Research Far East. He is a Professor Emeritus of The American University. He received his Ph.D. under John Slater at MIT and has been active in the physics of magnetoelastic phenomena and amorphous magnetism. In the first cycle of his life he was much involved in physics and public affairs.